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Contents

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Albert Einstein

Johannes Kepler.

In anxious and uncertain times like ours, when it is difficult to find pleasure in humanity and the course of human affairs, it is particularly consoling to think of the serene greatness of a Kepler. Kepler lived in an age in which the reign of law in nature was by no means an accepted certainty. How great must his faith in natural law have been, to have given him the strength to devote ten years of hard and patient work to the empirical investigation of the movement of the planets and the mathematical laws of that ~~move~~ movement, entirely on his own account, supported by no one and understood by very few! If we would honour his memory worthily, we must get as clear a picture as we can of his problem and the stages of its solution.

Copernicus had opened the eyes of the most intelligent to the fact that the best way to get a clear grasp of the apparent movements of the planents in the heavens was by regarding them as movements round the sun conceived as stationary. If the planets moved uniformly in a circle round the sun, it would have been comparatively easy to discover how these movements must look from the earth. Since, however, the phenomena to be dealt with were much more complicated than that, the task was a far harder one. The first thing to be done was to determine these movements empirically from the observations of Tycho

Brahe. Only then did it become possible to think about deducing the general laws which these movements satisfy.

To grasp how difficult a business it was even to find out about the actual movements of revolution, one has to realize the following. One can never see where a planet really is at any given moment, but only in what direction it is seen just then from the earth, which, however, is itself moving in an unknown manner round the sun. The difficulties thus seemed practically unsurmountable.

Kepler had to discover a way of bringing order into this chaos. To start with, he saw that it was necessary first to try to find out about the motion of the earth itself. This would have been simply ~~[illegible]~~ impossible if there had existed only the sun, the earth and the fixed stars, but no other planets. For in that case one could ascertain nothing empirically except how the direction of the straight line sun-earth changes in the course of the year (apparent movement of the sun with reference to the fixed stars). In this way it was possible to discover that these directions all lay in a plane stationary with reference to the fixed stars, at least with the accuracy of observation achieved in those days, when there were no telescopes. By this means it could also be ascertained in what manner the line sun-earth revolves round the sun. It turned out that the angular velocity of this motion went through a regular change in the course of the year. But this was not of much use, as it was still not known how the distance between the earth and the sun alters in the course of the year. It was only when these changes were known that the real shape

the earth's orbit and the manner in which it is described
were discovered.

Kepler found a marvellous way out of this dilemma. In the first place, it followed from the observations of the sun that the apparent path of the sun against the background of the fixed stars differed in speed at different times of the year, but that the angular velocity of this movement was always the same at the same point in the astronomical year, and therefore that the speed of rotation of the straight line earth-sun was always the same when it pointed to the same region of the fixed stars. It was thus legitimate to suppose that the earth's orbit was a self-enclosed one, described by the earth in the same way every year — which was by no means obvious 'a priori'. For the adherent of the Copernican system it was thus as good as certain that this must also apply to the orbits of the rest of the planets.

This certainly made things easier. But how to ascertain the real shape of the earth's orbit? Imagine a brightly shining lantern M somewhere in the plane of the orbit. We know that this lantern remains permanently in its place and thus forms a kind of fixed triangulation-point for determining the earth's orbit, a point which the inhabitants of the earth can take a sight on at any time of year. Let this lantern M be farther away from the sun than the earth. With the help of such a lantern it was possible to determine the earth's orbit, in the following way: —

First of all, in every year there comes a moment when the earth E lies exactly on the line joining the sun S and

Earth at
the lantern M. If at this moment we look from the
the lantern M, our line of sight will coincide with the line
SM (Sun-lantern). Suppose the latter to be marked in the
heavens. Now imagine the earth in a different position and at
a different time. Since the sun S and the lantern M can both be
seen from the earth, the angle at E in the triangle SEM is
known. But we also know the direction of SE in relation to
the fixed stars through direct solar observations, while the
direction of the line SM in relation to the fixed stars was
finally ascertained previously. But in the triangle SEM we
also know the angle at S. Therefore, with the base SM arbitrarily laid down on a sheet of paper, we can, in virtue of
our knowledge of the angles at E and S, construct the
triangle SEM. We might do this at frequent intervals
during the year; each time we should get on our piece
of paper a position of the earth E with a date attached
to it and a certain position in relation to the permanently fixed base SM. The earth's orbit would thereby
be empirically determined, apart from its absolute
size, of course.

But, you will say, where did Kepler get his lantern
M? His genius and Nature, benevolent in this case, gave it
to him. There was, for example, the planet Mars, and the
length of the <u>Martian year</u> - i.e., one rotation of Mars round
the sun - <u>was known</u>. It may happen one fine day that
the sun, the earth and Mars lie absolutely in the same
straight line. This position of Mars regularly recurs after
one, two, etc. Martian years, as Mars has a self-enclosed
orbit. At these known moments, therefore, SM always presents
the same base, while the earth is always at a different
point in its orbit. The observations of the sun and M

Mars
at these
These moments thus constitute a means of determining the true orbit of the earth, as Mars then plays the part of our imaginary lantern. Thus it was that Kepler discovered the true shape of the earth's orbit and the way in which the earth describes it, and we who come after — Europeans, Germans, or even Swabians — may well admire and honour him for it.

Now that the earth's orbit had been empirically determined, the true position and length of the line SE at any moment was known, and it was not so terribly difficult for Kepler to calculate the orbit and motions of the rest of the planets too from observations — at least in principle. It was nevertheless an immense work, especially considering the state of mathematics at the time.

Now came the second and no less arduous part of Kepler's life-work. The orbits were empirically known, but their laws had to be deduced from the empirical data. First he had to make a guess at the mathematical nature of the curve described by the orbit, and then try it out on a vast assemblage of figures. If it did not fit, another hypothesis had to be devised and again tested. After tremendous search, the conjecture that the orbit was an ellipse with the sun at one of its foci was found to fit the facts. Kepler also discovered the law governing the variation in speed during rotation, which is that the line sun-planet sweeps out equal areas in equal periods of time. Finally he also discovered that the square of the period of circulation round the sun varies as the cube of the major axes of the ellipse.

Our admiration for this splendid man is accompanied by another feeling of admiration and reverence, the object of which is no man but the mysterious harmony of

ancient times
nature into which we are born. As far back as ancient times people devised the lines exhibiting the simplest conceivable form of regularity. Among these, next to the straight line and the circle the most important were the ellipse and the hyperbola. We see the last two embodied – at least very nearly so – in the orbits of the heavenly bodies.

It seems that the human mind has first to construct forms independently before we can find them in things. Kepler's marvellous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone but only from the comparison of the inventions of the intellect with observed fact.

The Mechanics of Newton and their Influence on the Development of Theoretical Physics.

It is just two hundred years ago since Newton closed his eyes for the last time. It behoves us at such a moment to remember this brilliant genius, who determined the course of western thought, research and practice to an extent that nobody before or since his time can touch. Not only was he brilliant as an inventor of certain key methods, but he also had a unique command of the empirical material available in his day, and he was marvellously inventive as regards mathematical and physical methods of proof in individual cases. For all these reasons he deserves our deepest reverence. The figure of Newton has, however, an even greater importance than his genius warrants from the fact that destiny placed him at a turning point in the history of the human intellect. To see this vividly, we have to

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remind ourselves that before Newton there existed no self-contained system of physical causality which was capable of representing any of the deeper features of the empirical world.

No doubt the great materialists of ancient Greece had insisted that all material events should be traced back to a strictly regular series of atomic movements, without admitting any living creature's will as an independent cause. And no doubt Descartes had in his own way taken up this quest again. But it remained a bold ambition, the problematical ideal of a school of philosophers. Actual results of a kind to support the belief in the existence of a complete chain of physical causation hardly existed before Newton.

Newton's object was to answer the question, Is there such a thing as a simple rule by which one can calculate the movements of the heavenly bodies in our planetary system completely, when the state of motion of all these bodies at one moment is known? Kepler's empirical laws of planetary movement, deduced from Tycho Brahe's observations, confronted him, and demanded explanation'. These laws gave, it is true, a complete answer to the question how the planets move round the sun (the elliptical shape of the orbit, equal areas in equal times, the relation between the

{ ' To-day everybody knows what prodigious industry was needed to discover these laws from the empirically ascertained orbits. But few pause to reflect on the brilliant methods by which Kepler deduced the real orbits from the apparent ones — i.e. from the movements as they were observed from the earth. }

major axes and the period of circulation round the sun, but they did not satisfy the demand for causality. They are three logically independent rules, revealing no inner connection with each other. The third law cannot simply be transferred quantitatively to other central bodies than the sun (there is, for example, no relation between the rotatory period of a planet round the sun and that of a moon round the planet). The most important point, however, is this: these laws are concerned with the movement as a whole, and not with the question how the state of motion of a system gives rise to that which immediately follows it in time; they are, as we should say now, integral and not differential laws.

The differential law is the only form which completely satisfies the modern physicist's demand for causality. The clear conception of the differential law is one of Newton's greatest intellectual achievements. It was not merely the notion that was needed but also a mathematical formalism which existed in its rudiments but had to acquire a systematic form. Newton found this also in the differential and the integral calculus. We need not consider the question here whether Newton hit upon the same mathematical methods independently of Leibnitz or not. In any case it was absolutely necessary for Newton to perfect them, since they alone could provide him with the means of expressing his ideas.

Galileo had already made a considerable advance towards a knowledge of the law of motion. He discovered the law of inertia and the law of bodies falling freely in the

gravitational
gravitational field of the earth: namely, that a mass, (more accurately, mass-point) which is unaffected by other masses, moves uniformly and in a straight line. The vertical speed of a free body in the gravitational field increases uniformly with the time. It may seem to us to-day to be but a short step from Galileo's discoveries to Newton's law of motion. But it should be observed that both the above statements refer in their form to the motion as a whole, whereas Newton's law of motion provides an answer to the question, How does the state of motion of a mass-point behave in an infinitely short time under the influence of an external force? It was only by considering what takes place during an infinitely short time (the differential law) that Newton reached a formula which applies to all motion whatsoever. He took the concept of force from statics, which had already reached a high stage of development. The connection between force and acceleration was only made possible for him by the introduction of the new concept of mass, which was supported, strange to say, by an illusory definition. We are so accustomed to-day to the creation of concepts corresponding to differential quotients that we can hardly grasp now what a remarkable power of abstraction it needed to reach the general differential law by a crossing of two frontiers in the course of which the concept of mass had in addition to be invented.

But a causal conception of motion was still far from being achieved. For the motion was only determined by the equation of motion in cases where the force was given. Inspired no doubt by the uniformity of planetary motions, Newton conceived the idea that the force operating on a

situated at
mass was determined by the position of all masses situated at a sufficiently small distance from the mass in question. It was not till this connection was established that a completely causal conception of motion was achieved. How Newton, starting from Kepler's laws of planetary motion, performed this task for gravitation and so discovered that the kinetic forces acting on the stars and gravity were of the same nature, is well known. It is the combination of the Laws of Motion with the Law of Attraction which constitutes that marvellous edifice of thought which makes it possible to calculate the past and future states of a system from the state obtaining at one particular moment, in so far as the events take place under the influence of the forces of gravity alone. The logical completeness of Newton's conceptual system lay in this, that the only things that figure as causes of the acceleration of the masses of a system are these masses themselves.

On the strength of the basis here briefly sketched, Newton succeeded in explaining the motions of the planets, moons and comets down to the smallest details, as well as the tides and the precessional movement of the earth — a deductive achievement of unique magnificence. The discovery that the cause of the motions of the heavenly bodies is identical with the gravity with which we are so familiar from everyday life must have been particularly impressive.

But the importance of Newton's achievement was not confined to the fact that it created a workable and logically satisfactory basis for the actual science of mechanics; up to the end of the nineteenth century it formed the programme of every worker in the field of theoretical physics. All physical events were to be traced back to masses which are subject to Newton's laws of motion. The laws of force simply had to be widened and

adapted
adapted to the type of event under consideration. Newton himself tried to apply this scheme to optics, assuming light to consist of inert corpuscles. Even the wave theory of light made use of Newton's law of motion, after it had been applied to the mass of a continuum. Newton's equations of motion were the sole basis of the kinetic theory of heat, which not only prepared people's minds for the discovery of the law of the conservation of energy but also led to a theory of gases which has been confirmed down to the last detail, and a more profound view of the nature of the second law of thermodynamics. The development of electricity and magnetism has proceeded right down to our own day along Newtonian lines (electrical and magnetic substance, forces acting at a distance). Even the revolution in electrodynamics and optics brought about by Faraday and Clerk Maxwell, which formed the first great fundamental advance in theoretical physics since Newton, took place entirely under the aegis of Newton's ideas. Clerk Maxwell, Boltzmann and Lord Kelvin never wearied of tracing the electro-magnetic fields and their reciprocal dynamic actions back to the mechanical action of hypothetical continuous media possessing mass. As a result, however, of the ~~p~~ hopelessness or at any rate the lack of success of those efforts, a gradual revolution in our fundamental notions has taken place since the end of the nineteenth century; theoretical physics have outgrown the Newtonian frame which gave stability and intellectual guidance to science for nearly two hundred years.

Newton's fundamental principles were so satisfactory from the logical point of view that the impetus to overhaul them could only spring from the imperious demands of empirical fact. Before I go into this I must insist that Newton himself

was better aware of the weaknesses inherent in his individual edifice than the generations of scientists which followed him. This fact has always roused my respectful admiration, and I should like, therefore, to dwell on it for a moment.

(1) In spite of the fact that Newton's ambition to represent his system as necessarily conditioned by experience and to introduce the smallest possible number of concepts not directly referable to empirical objects is everywhere evident, he sets up the concept of absolute space and absolute time, for which he has often been criticised in recent years. But in this point Newton is particularly consistent. He had realised that observable geometrical magnitudes (distances of material points from one another) and their course in time do not completely characterise motion in its physical aspects. He proves this in the famous experiment with the rotating vessel of water. Therefore, in addition to masses and their temporally variable distances, there must be something else that determines motion. That "something" he takes to be relation to "absolute space". He is aware that space must possess a kind of physical reality if his laws of motion are to have any meaning a reality of the same sort as material points and the intervals between them.

This clear realisation of this reveals both Newton's wisdom and also a weak side to his theory. For the logical structure of the latter would undoubtedly be more satisfactory without this shadowy concept; in that case only things whose relations to perception are perfectly clear (mass-points, distances) would enter into the laws.

(2) The introduction of forces acting directly and instantaneously at a distance into the representation of the effects of gravity is not in keeping with the character

of

most of the processes familiar to us from everyday life. Newton meets this objection by pointing to the fact that his law of reciprocal gravitation is not supposed to be a final explanation but a rule derived by induction from experience.

(3) Newton's doctrine provided no explanation for the highly remarkable fact that the weight and the inertia of a body are determined by the same quantity (its mass). The remarkableness of this fact struck Newton himself.

None of these three points can rank as a logical objection to the theory. In a sense they merely represent unsatisfied desires of the scientific spirit in its struggle for a complete and unitary penetration of natural events by thought.

Newton's doctrine of motion, considered as the key idea of the whole theoretical physics, received its first shock from Clerk Maxwell's theory of electricity. It became clear that the reciprocal actions between bodies due to electric and magnetic forces were effected, not by forces operating instantaneously at a distance, but by processes which are propagated through space at a finite speed. Faraday conceived a new sort of real physical ~~unity~~ entity, namely the "field", in addition to the mass-point and its motion. At first people tried, clinging to the mechanical mode of thought, to look upon it as a mechanical condition (motion or force) of a hypothetical medium by which space was filled up (the ether). But when this interpretation refused to work in spite of the most obstinate efforts, people gradually got used to the idea of regarding the "electro-magnetic field" as a final irreducible constituent of physical reality. We have H. Hertz to thank for definitely freeing the conception of the field from all encumbrances derived from the conceptual armoury of

mechanics, and H.A. Lorentz for freeing it from a motion substratum, according to the latter the only thing left to act as a substratum for the field was physical empty space (or ether), which even in the mechanics of Newton had not been destitute of all physical functions. By the time this point was reached, nobody any longer believed in immediate momentary action at a distance, not even in the sphere of gravitation, even though no field-theory of the latter had been clearly sketched out owing to lack of sufficient factual knowledge. The development of the theory of the electro-magnetic field — once Newton's hypothesis of forces acting a distance had been abandoned — led to the attempt to explain the Newtonian law of motion on electro-magnetic lines or alternatively to replace it by a more accurate one based on the field-theory. Even if these efforts did not meet with complete success, still the fundamental concepts of mechanics had ceased to be looked upon as fundamental constituents of the physical universe.

The theory of Clerk Maxwell and Lorentz led inevitably to the special theory of relativity, which ruled out the existence of forces acting at a distance, with the resulting destruction of the notion of absolute simultaneity. This theory made it clear that mass is not a constant quantity but depends on the energy-content — is indeed equivalent to it. It also showed that Newton's law of motion was only to be regarded as a limiting law valid for small velocities; in its place it put a new law of motion in which the speed of light in vacuo figures as the critical velocity.

The general theory of relativity formed the last step in the development of the programme of the field-theory. Quantitatively it modified Newton's theory only slightly, but all the more profoundly for that qualitatively. Inertia,

gravitation
~~-vitation~~, and the metrical behaviour of bodies and clocks were reduced to a single field quality, this field itself was again placed in dependence on the bodies (generalisation of Newton's laws of gravity or the field-law corresponding to it, as formulated by Poisson). Space and time were thereby divested not of their reality but of their causal absoluteness (absoluteness affecting but not affected) which Newton had been compelled to ascribe to them in order to be able to give expression to the laws then known. The generalised law of inertia takes over the function of Newton's law of motion. This short account is enough to show how the elements of the Newtonian theory passed over into the general theory of relativity, whereby the three defects above mentioned were overcome. It looks as if the law of motion could be deduced from the field-law corresponding to the Newtonian law of force. Only when this goal has been completely reached will it be possible to talk about a pure field-theory.

In a more formal sense also Newton's mechanics prepared the way for the field-theory. The application of Newton's mechanics to continuously distributed masses led inevitably to the discovery and application of partial differential equations, which in their turn first provided the language ~~of~~ for the laws of the field-theory. In this formal respect Newton's conception of the differential law constitutes the first decisive step in the development which followed.

The whole evolution of our ideas about the processes of nature, with which we have been concerned so far, might be regarded as an organic development of Newton's ideas. But while the process of perfecting the field-theory was still in full swing, the facts of heat-radiation, the spectra, radioactivity etc., revealed a limit to the serviceableness of the whole intellectual system which to-day still seems to

us absolutely insuperable in spite of immense successes at certain points. Many physicists maintain — and there are weighty arguments in their favour — that in the face of these facts not merely the differential law but the law of causation itself — hitherto the fundamental postulate of all natural science — has collapsed. Even the possibility of a spatio-temporal construction, which can be unambiguously co-ordinated with physical events, is denied. That a mechanical system is permanently susceptible only of discrete energy-values or states — as experience, so to speak, directly shows — seems at first sight hardly deducible from a field-theory which operates with differential equations. The De Broglie-Schrödinger method, which has in a certain sense the character of a field-theory, does indeed deduce the discreteness of energy states, in astonishing agreement with empirical fact, on the basis of differential equations operating with a kind of resonance-theory, but it has to do without a localisation of the mass-particles and without strictly causal laws. Who would presume to-day to decide the question whether the law of causation and the differential law, these ultimate premisses of the Newtonian view of nature, must definitely be given up?

Clerk Maxwell's Influence on the Evolution of the Idea of Physical Reality.

The belief in an external world independent of the perceiving subject is the basis of all natural science. Since, however, sense perception only gives information of this external world or of "physical reality" indirectly, we can only grasp the latter by speculative means. It follows from this that our notions of physical reality can never be final. We must always be ready to change these notions — that is

to say, the axiomatic substructure of physics — in order to do justice to perceived facts in the most logically perfect way. Actually, a glance at the development of physics shows that it has undergone far-reaching changes in the course of time.

The greatest change in the axiomatic substructure of physics — in other words, of our conception of the structure of reality — since Newton laid the foundation of theoretical physics was brought about by Faraday's and Clerk Maxwell's work on electro-magnetic phenomena. We will try in what follows to make this clearer, keeping both earlier and latter developments in sight.

According to Newton's system, physical reality is characterised by the concepts of time, space, material point, and force (= reciprocal action of material points). Physical events, in Newton's view, are to be regarded as the motions, governed by fixed laws, of material points in space. The material point is our only mode of representing reality, when dealing with changes taking place in it. Perceptible bodies are obviously responsible for the concept of the material point; people conceived it as an analogue of mobile bodies, stripping these of the characteristics of extension, form, orientation in space and all "inward" qualities, leaving only inertia and translation and adding the concept of force. The material bodies, which had led psychologically to our formation of the concept of the "material point", had now themselves to be regarded as systems of material points. It should be noted that this theoretical scheme is in essence an atomistic and mechanistic one. All happenings were to be interpreted purely mechanically — that is to say, simply as motions of material points according to Newton's law of motion.

The most unsatisfactory side of this system (apart from the difficulties involved in the concept of "absolute space" which have been raised once more just recently) lay in its description of light, which Newton also conceived, in accordance with his system, as composed of material points. Even at that time the question, What in that case becomes of the material points of which light is composed, when the light is absorbed? was already a burning one. Moreover, it is unsatisfactory in any case to introduce into the discussion material points of quite a different sort, which have to be postulated for the purpose of representing ponderable matter and light respectively. Later on corpuscles of electricity were added to these, making a third kind, again with completely different characteristics. It was, further, a fundamental weakness that the forces of reciprocal action, by which events are determined, had to be assumed hypothetically in a perfectly arbitrary way. Yet this conception of the real accomplished much: how came it that people felt themselves impelled to forsake it?

In order to put his system into mathematical form at all, Newton had to devise the concept of differential quotients and propound the laws of motion in the form of total differential equations — perhaps the greatest advance in thought that a single individual was ever privileged to make. Partial differential equations were not necessary for this purpose, nor did Newton make any systematic use of them; but they were necessary for the formulation of the mechanics of deformable bodies; this is connected with the fact that in these problems the question of how bodies are supposed to be constructed out of material points was of no importance to begin with.

Thus the partial differential equation entered theoretical physics as a handmaid, but has gradually become mistress.

This began in the nineteenth century when the wave-theory of light established itself under the pressure of observed fact. Light in empty space was explained as a matter of vibrations of the ether, and it seemed idle at that stage, of course, to look upon the latter as a conglomeration of material points. Here for the first time the partial differential equation appeared as the natural expression of the primary realities of physics. In a particular department of theoretical physics the ~~continuous~~ continuous field thus appeared side by side with the material point as the representative of physical reality. This dualism remains even to-day, disturbing as it must be to every orderly mind.

If the idea of physical reality had ceased to be purely atomic, it still remained for the time being purely <u>mechanistic</u>; people still tried to explain all events in terms of the motion of inert masses; indeed no other way of looking at things seemed conceivable. Then came the great change, which will be associated for all time with the names of Faraday, Clerk Maxwell and Hertz. The lion's share in this revolution fell to Clerk Maxwell. He showed that the whole of what was then known about light and electro-magnetic phenomena was expressed in his well-known double system of differential equations, in which the electric and the magnetic fields appear as the dependent variables. Maxwell did, indeed, try to explain, or justify, these equations by intellectual constructions.

But he made use of several such constructions at the same time and took none of them really seriously, so that the equations alone appeared as the essential thing and the strength of the fields as the ultimate entities, not to be reduced to anything else. By the turn of the century the conception of the

electro-magnetic field as an ultimate entity had been fully accepted and serious thinkers had abandoned the belief in the justification, or the possibility, of a mechanical explanation of Clerk Maxwell's equations. Before long they were, on the contrary, actually trying to explain material points and their inertia on field-theory lines with the help of Maxwell's theory, an attempt which did not, however, meet with complete success.

Neglecting the important individual results which Clerk Maxwell's life-work produced in several main departments of physics, and concentrating on the changes wrought by him in our conception of the nature of physical reality, we may say this:— Before Clerk Maxwell physical reality was conceived — in so far as it was intended to represent events in nature — as made up of material points, whose changes consist exclusively of motions which are subject to partial differential equations. After Maxwell they conceived physical reality as represented by continuous fields, not mechanically explicable, which are subject to partial differential equations. This change in the conception of reality is the most profound and fruitful one that has come to physics since Newton; but it has at the same time to be admitted that the programme has not yet been completely carried out by any means. The successful systems of physics which have been evolved since rather represent compromises between these two schemes, which for that very reason bear a provisional, logically incomplete character, although they may have achieved great advances in certain particulars.

The first of these that calls for mention is Lorentz's theory of electrons, in which the field and

the electrical corpuscles appear side by side as elements of equal value for the comprehension of reality. Next come the special and general theories of relativity, which, though based entirely on ideas connected with the field-theory, have so far been unable to avoid the independent introduction of material points and total differential equations.

The last and most successful creation of theoretical physics, namely quantum-mechanics, differs fundamentally from both the schemes which we will for the sake of brevity call the Newtonian and Maxwellian. For the quantities which figure in its laws make no claim to describe physical reality itself, but only the probabilities of the occurrence of a physical reality that we have in view. Dirac, to whom, in my opinion, we owe the most logically complete exposition of this theory, rightly points out that it would probably be difficult, for example to give a theoretical description of a photon such as would give enough information to enable one to decide whether it will pass a polariser placed (obliquely) in its way or not.

I am still inclined to view that physicists will not in the long run content themselves with this sort of indirect description of the real, even if the theory can eventually be adapted to the postulate of general relativity in a satisfactory manner. We shall then, I feel sure, have to return to the attempt to carry out the programme which may properly be described as the Maxwellian — namely, the description of physical reality in terms of fields which satisfy partial differential equations without singularities.

Niels Bohr.

When a later generation comes to write the history of the progress made in physics in our time, it will have to connect one of the most important advances ever made in our knowledge of the nature of the atom with the name of Niels Bohr. It was already known that classical mechanics break down in relation to the ultimate constituents of matter, also that atoms consist of positively charged nuclei which are surrounded by a layer of atoms of relatively lose texture. But the structure of the spectra, which was to a large extent known empirically, was so profoundly different from what was to be expected on our older theories that nobody could find a convincing theoretical interpretation of the observed uniformities. Thereupon Bohr in the year 1913 devised an interpretation of the simplest spectra on quantum-theory lines, for which he in a short time produced such a mass of quantitative confirmation that the boldly selected hypothetical basis of his speculations soon became a mainstay for the physics of the atom. Although less than ten years have passed since Bohr's first discovery, the system conceived in its main features and largely worked out by him already dominates both physics and chemistry so completely that all earlier systems seem to the expert to date from a long vanished-age. The theories of X-ray spectra, of visible spectra, and of the periodic system of the elements are primarly based on the ideas of Bohr. What is so marvellously attractive about Bohr as a scientific thinker is his rare blend of boldness and caution; seldom has anyone possessed such an intuitive grasp of hidden things combined with such a strong critical sense. With all his knowledge of the details, his eye is immovably fixed on the underlying principle. He is unquestionably one of the greatest discoverers of our age in the scientific field.

On the Theory of Relativity.

An Address in London.

It is a particular pleasure to me to have the privilege of speaking in the capital of the country from which the most important fundamental notions of theoretical physics have issued. I am thinking of the theory of mass motion and gravitation which Newton gave us and of the concept of the electromagnetic field, by means of which Faraday and Clerk Maxwell put physics on a new basis. The theory of relativity may indeed be said to have put a sort of finishing touch to the ~~mighty~~ mighty intellectual edifice of Maxwell and Lorentz, inasmuch as it seeks to extend field-physics to all phenomena, gravitation included.

Turning to the theory of relativity itself, I am anxious to draw attention to the fact that this theory is not speculative in origin; it owes its invention entirely to the desire to make physical theory fit observed fact as well as possible. We have here no revolutionary act but the natural continuation of a line that can be traced through centuries. The abandonment of a certain concept connected with space, time and motion hitherto treated as fundamental must not be regarded as arbitrary but only as conditioned by observed facts.

The law of the constant velocity of light in empty space, which has been confirmed by the development of electrodynamics and optics, and that of the equal legitimacy of all inertial systems (special principle of relativity), which was proved in a particularly incisive manner by Michelson's famous experiment, between them made it necessary, in the first place, that the concept of time should be made

relative, each inertial system being given its own spatial
special time. As this notion was developed it became clear that the connection between immediate experience on one side and co-ordinates and time on the other had hitherto not been thought out with sufficient precision. It is in general one of the essential features of the theory of relativity that it is at pains to work out the relations between general concepts and empirical facts more precisely. The fundamental principle here is that the justification for a physical concept lies exclusively in its clear and unambiguous relation to facts that can be experienced.

According to the special theory of relativity, spatial co-ordinates and time still have an absolute character in so far as they are directly measurable by stationary clocks and bodies. But they are relative in so far as they depend on the state of motion of the selected inertial system. According to the special theory of relativity the four-dimensional continuum formed by the union of space and time retains the absolute character which, according to the earlier theory, belonged to both space and time separately (Minkowski). The influence of motion (relative to the co-ordinate system) on the form of bodies and on the motion of clocks, also the equivalence of energy and inert mass, follow from the interpretation of co-ordinates and time as products of measurement.

The general theory of relativity owes its existence in the first place to the empirical fact of the numerical equality of the inertial and gravitational mass of bodies, for which fundamental fact, classical mechanics provided

~~no~~ interpretation. Such an interpretation is arrived at by an ~~[illegible]~~ extension of the principle of Relativity to co-ordinate systems accelerated ~~by~~ relatively to one another. The introduction of co-ordinate systems accelerated relatively to inertial systems involves the appearance of gravitational fields relative to the latter. As a result of this, the general theory of relativity, which is based on the equality of inertia and weight, provides a theory of the gravitational field.

The introduction of co-ordinate systems accelerated relatively to each other as equally legitimate systems, such as they appear conditioned by the identity of inertia and weight, leads, in conjunction with the results of the special theory relativity, to the conclusion that the laws governing the occupation of space by solid bodies, when gravitational fields are present, do not correspond to the laws of Euclidean geometry. An analogous result follows from the motion of clocks. This brings us to the necessity for yet another generalisation of the theory of space and time, because the direct interpretation of space and time co-ordinates by means of measurements obtainable with measuring rods and clocks now breaks down. That generalisation of metric, which had already been accomplished in the sphere of pure mathematics by the researches of Gauss and Riemann, is essentially based on the fact that the metric of the special theory of relativity can still claim validity for small areas in general case too.

The process of development here sketched strips the space-time co-ordinates of all independent reality. The metrically real is now only given through the combination of the space-time co-ordinates

with the mathematical quantities which describe the gravitational field.

There is yet another factor underlying the evolution of the general theory of relativity. As Ernst Mach insistently pointed out, the Newtonian theory is unsatisfactory in the following respect:- If one considers motion from the purely descriptive, not from the causal point of view, it only exists as relative motion of things with respect to one another. But the acceleration which figures in Newton's equations of motion is unintelligible if one starts with the concept of relative motion. It compelled Newton to invent a physical space in relation to which acceleration was supposed to exist. This introduction ad hoc of the concept of absolute space, while logically unexceptionable, nevertheless seems unsatisfactory. Hence the attempt to alter the mechanical equations in such a way that the inertia of bodies is traced back to relative motion on their part not as against absolute space but as against the totality of other ponderable bodies. In the state of knowledge then existing his attempt was bound to fail.

The posing of the problem seems, however, entirely reasonable. This line of argument imposes itself with considerably enhanced force in relation to the general theory of relativity, since, according to that theory, the physical properties of space are affected by ponderable matter. In my opinion, the general theory of relativity can only solve this problem satisfactorily if it regards the world as spatially self-enclosed. The mathematical results of the theory force one to this view, if one believes that the mean density of ponderable matter in the world possesses some ultimate value, however small.

What is the Theory of Relativity?

I gladly accede to the request of your colleague to write something for 'The Times' on relativity. After the lamentable breakdown of the old active intercourse between men of learning, I welcome this opportunity of expressing my feelings of joy and gratitude towards the astronomers and physicists of England. It is thoroughly in keeping with the great and proud traditions of scientific work in your country that eminent scientists should have spent much time and trouble, and your scientific institutions have spared no expense, to test the implications of a theory which was completed and published during the war in the land of your enemies. Even though the investigation of the influence of the gravitational field of the sun on light rays is a purely objective matter, I cannot forbear to express my personal thanks to my English colleagues for their work; for without it I could hardly have lived to see the most important implication of my theory tested.

We can distinguish various kinds of theories in Physics. Most of them are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out. Thus the kinetic theory of gases seeks to reduce mechanical, thermal and diffusional processes to movements of molecules — i.e. to build them up out of the hypothesis of molecular motion. When we say that we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question.

Along with this most important class of theories there exists a second, which I will call "principle-theories". These employ the analytic, not the synthetic, method. The elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. Thus the science of thermodynamics seeks by analytical means to deduce necessary connections, which separate events have to satisfy, from the universally experienced fact that perpetual motion is impossible.

The advantages of the constructive theory are completeness, adaptability and clearness, those of the principle-theory are logical perfection and security of the foundations.

The theory of relativity belongs to the latter class. In order to grasp its nature, one needs first of all to become acquainted with the principles on which it is based. Before I go into these, however, I must observe that the theory of relativity resembles a building consisting of two separate storeys, the special theory and the general theory. The special theory, on which the general theory rests, applies to all physical phenomena with exception of gravitation; the general theory provides the law of gravitation and its relations to the other forces of nature.

It has, of course, been known since the days of the ancient greeks that in order to describe the movement of a body, a second body is needed to which the movement of the first is reffered. The movement of a vehicle is considered in reference to the earth's surface, that of a planet to the totality of the visible fixed stars. In physics the body to which events are

spatially referred is called the co-ordinate system. The laws of mechanics of Galileo and Newton, for instance, can only be formulated with the aid of a co-ordinate system.

The state of motion of the co-ordinate system may not, however, be arbitrarily chosen, if the laws of mechanics are to be valid (it must be free from rotation and acceleration). A co-ordinate system which is admitted in mechanics is called an "inertial system". The state of motion of an inertial system is according to mechanics not one that is determined unambiguously by nature. On the contrary, the following definition holds good :— a co-ordinate system that is moving uniformly and in a straight line relatively to an inertial system is likewise an inertial system. By the "special principle of relativity" is meant the generalisation of this definition to include any natural event whatever: thus, every universal law of nature which is valid in relation to a co-ordinate system C, must also be valid, as it stands, in relation to a co-ordinate system C', which is in uniform translatory motion relatively to C.

The second principle, on which the special theory of relativity rests, is the "principle of the constant velocity of light in vacuo". This principle asserts that light in vacuo always has a definite velocity of propagation, independent of the state of motion of the observer or of the source of light. The confidence which physicists place in this principle springs from the successes achieved by the electro-dynamics of Clerk Maxwell and Lorentz.

Both the above-mentioned principles are powerfully supported by experience, but appear not to be logically reconcilable. The special theory of relativity finally succeeded in reconciling them logically by a modification of kinematics — i.e., of the doctrine of the laws relating to space and time (from the point of view of physics). It became clear that to speak of the simultaneity of two events had no

meaning except in relation to a given co-ordinate system, i
the shape of measuring devices and the speed at which clocks move depend on their state of motion with respect to the co-ordinate system.

But the old physics, including the laws of motion of Galileo and Newton, did not fit in with the suggested relativist kinematics. From the latter general mathematical conditions issued, to which natural laws had to conform, if the above-mentioned two principles were really to apply. To these physics had to be adapted. In particular, scientists arrived at a new of motion for (rapidly moving) mass-points, which was admirably confirmed in the case of electrically charged particles. The most important upshot of the special theory of relativity concerned the inert mass of corporeal systems. It turned out that the inertia of a system necessarily depends on its energy-content, and this led straight to the notion that inert mass is simply latent energy. The principle of the conservation of mass lost its independence and became fused with that of the conservation of energy.

The special theory of relativity, which was simply a systematic development of the electro-dynamics of Clerk Maxwell and Lorentz, pointed beyond itself, however. Should the independence of physical laws of the state of motion of the co-ordinate system be restricted to the uniform translatory motion of co-ordinate systems in respect to each other? What has nature to do with our co-ordinate systems and their state of motion? If it is necessary for the purpose of describing nature to make use of a co-ordinate system arbitrarily introduced by us, then the choice of its state of motion ought to be subject to no restriction; the laws ought to be entirely independent of this choice (general principle of relativity).

The establishment of this general principle of relativity

is made easier by a fact of experience that has long been known, namely that the weight and the inertia of a body are controlled by the same constant (Equality of inertial and gravitational mass). Imagine a co-ordinate system which is rotating uniformly with respect to an inertial system in the Newtonian manner. The centrifugal forces which manifest themselves in relation to this system must, according to Newton's teaching, be regarded as effects of inertia. But these centrifugal forces are, exactly like the forces of gravity, proportional to the masses of the bodies. Ought it not to be possible in this case to regard the co-ordinate system as stationary and the centrifugal forces as gravitational forces? This seems the obvious view, but classical mechanics forbid it.

This hasty consideration suggests that a general theory of relativity must supply the laws of gravitation, and the consistent following-up of the idea has justified our hopes.

But the path was thornier than one might suppose, because it demanded the abandonment of Euclidean geometry. ~~That~~ That is to say, the laws according to which fixed bodies may be arranged in space do not completely accord with the spatial attributed to bodies by Euclidean geometry. This is what we mean when we talk of the "Curvature of space". The fundamental concepts of the "straight line", the "plane" etc. thereby lose their precise significance in physics.

In the general theory of relativity the doctrine of space and time, or kinematics, no longer figures as a fundamental independent of the rest of physics. The geometrical behaviour of bodies and the motion of clocks rather depend on gravitational fields, which in their turn are produced by matter.

The new theory of gravitation diverges considerably, as regards

principles, from Newton's theory. But its practical results agree with those of Newton's theory that it is difficult to find criteria for distinguishing them which are accessible to experience. Such have been discovered so far:—

(1) In the revolution of the ellipses of the planetary orbits round the sun (confirmed in the case of Mercury).

(2) In the curving of light-rays by the action of gravitational fields (confirmed by the English photographs of eclipses).

(3) In a displacement of the spectral lines towards the red end of the spectrum in the case of light transmitted to us from stars of considerable magnitude (unconfirmed so far)[1].

The chief attraction of the theory lies in its logical completeness. If a single one of the conclusions drawn from it proves wrong, it must be given up; to modify it without destroying the whole structure seems to be impossible.

Let no one suppose, however, that the mighty work of Newton can really be superseded by this or any other theory. His great and lucid ideas will retain their unique significance for all time as the foundation of our whole modern conceptual structure in the sphere of natural philosophy.

Addendum. Some of the statements in your paper concerning my life and person owe their origin to the lively imagination of the writer. Here is yet another application of the principle of relativity for the delectation of the reader:– To-day I am described in Germany as a "German savant", and in England as a "Swiss Jew". Should it ever be my fate to be represented a 'bête noire', I should, on the contrary, become a "Swiss Jew" for the Germans and a "German savant" for the English.

[[1] Editor's note: This criterion has also been confirmed in the meantime.]

The Problem of Space, Ether and the Field in Physics.

Scientific thought is a development of pre-scientific thought. As the concept of space was already fundamental in the latter, we must begin with the concept of space in pre-scientific thought. There are two ways of regarding concepts, both of which are necessary to understanding. The first is that of logical analysis. It answers the question, How do concepts and judgements depend on each other? In answering it we are on comparatively safe ground. It is the security by which we are so impressed in mathematics. But this security is purchased at the price of emptiness of content. Concepts can only acquire content when they are connected, however indirectly, with sensible experience. But no logical investigation can reveal this connection; it can only be experienced. And yet it is this connection that determines the cognitive value of systems of concepts.

Take an example. Suppose an archaeologist belonging to a latter culture finds a text-book of Euclidean geometry without diagrams. He will dis-cover how the words "point", "straight line", "plane", are used in the propositions. He will also see how the latter are deduced from each other. He will even be able to frame new propositions according to the known ~~the~~ rules. But framing of these propositions will remain an empty word-game for him, as long as "point", "straight line", "plane" etc. "convey nothing" to him. Only when they do convey something will geometry possess any real content for him. The same will be true of analytical ~~mechanics~~ mechanics, and indeed of any exposition of the logically deductive sciences.

What does this talk of "straight line", "point", "intersection etc." conveying something to one" mean? It means that one can point to the parts of sensible experience to which those words refer.

This extra-logical problem is the essential problem, which the archaeologist will only be able to solve intuitively, by examining his experience and seeing if he can discover anything which corresponds to those primary terms of the theory and the axioms laid down for them. Only in this sense can the question of the nature of a conceptually presented entity be reasonably raised.

With our pre-scientific concepts we are very much in the position of our archaeologist in regard to the ontological problem. We have, so to speak, forgotten what features in the world of experience caused us to frame those concepts, and we have great difficulty in representing the world of experience to ourselves without the spectacles of the old-established conceptual interpretation. There is the further difficulty that our language is compelled to work with words which are inseparably connected with those primitive concepts. These are the obstacles which confront us when we try to describe the essential nature of the pre-scientific concept of space.

One remark about concepts in general, before we turn to the problem of space: Concepts have reference to sensible experience, but they are never, in a logical sense, deducible from them. For this reason I have never been able to understand the quest of the a priori in the Kantian sense. In any ontological question, the only possible procedure is to seek out those characteristics in the complex of sense experiences to which the concepts refer.

Now as regards the concept of space: this seems to presuppose the concept of the solid object. The nature of the complexes and sense-impressions which are probably responsible for that concept has often been described. The correspondence between certain visual and <u>tactile</u> impressions, the fact that they can

be continuously followed out through time, and that the impressions can be repeated at any moment (taste, sight), are some of those characteristics. Once the concept of the solid object is formed in connection with the experiences just mentioned – which concept by no means presupposes that of space ~~and~~ or spatial relation – the desire to get an intellectual grasp of the relations of such solid bodies is bound to give rise to concepts which correspond to their spatial relations. Two solid objects may touch one another or be distant from one another. In the latter case, a third body can be inserted between them without altering them in any way; in the former not. These spatial relations are obviously real in the same sense as the bodies themselves. If two bodies are of equal value for the filling of one such interval, they will also prove of equal value for the filling of other intervals. The interval is thus shown to be independent of the selection of any special body to fill it; the same universally true of spatial relations. It is plain that this independence, which is a principal condition of the usefulness of framing purely geometrical concepts, is not necessarily a priori. In my opinion, this concept of the interval, detached as it is from the selection of any special body to occupy it, is the starting-point of the whole concept of space.

Considered, then, from the point of view of sense experience, the development of the concept of space seems, after these brief indications, to conform to the following schema – solid body; spatial relations of solid bodies; interval; space. Look-at in this way, space appears as something real in the same sense as solid bodies.

It is clear that the concept of space as a real thing already existed in the extra-scientific conceptual world. Euclid's mathematics, however, knew nothing of this concept as such;

they confined themselves to the concept of the object, and the spatial relations between objects. The point, the plane, the straight line, length, are solid objects idealised. All spatial relations are reduced to those of contact (the intersection of straight lines and planes, points lying on straight lines, etc.). Space as a continuum does not figure in the conceptual system at all. This concept was first introduced by Descartes, when he described the point-in-space by its co-ordinates. Here for the first time geometrical figures appear, up to a point, as parts of infinite space, which is conceived as a three-dimensional continuum.

The great superiority of the Cartesian ~~tot~~ treatment of space is by no means confined to the fact that it applies analysis to the purposes of geometry. The main point, seems rather to be this:— The geometry of the Greeks prefers certain figures (the straight line, the plane) in geometrical descriptions; other figures (eg., the ellipse) are only accessible to it because it constructs or defines them with the help of the point, the straight line and the plane. In the Cartesian treatment on the other hand, all surfaces are, in principle, equally ~~essential~~ represented, without any arbitrary preference for linear figures in the construction of geometry.

In so far as geometry is conceived as the science of laws governing the mutual relations of practically rigid bodies in space, it is to be regarded as the oldest branch of physics. This science was able, as I have already observed, to dispense with the concept of space as such; the ideal corporeal forms — point, straight line, plane, length — being sufficient for its needs. On the other hand, space as a whole, as conceived by Descartes, was absolutely necessary to Newtonian physics. For dynamics cannot manage with the concepts of the mass-point and the (temporally variable) distance between mass-points, alone.

Newton's equations of motion the concept of acceleration plays a fundamental part, which cannot be defined by the temporally variable intervals between points alone. Newton's acceleration is only thinkable or definable in relation to space as a whole. Thus to the geometrical reality of the concept of space a new inertia-determining function of space was added. When Newton described space as absolute, he no doubt meant this real significance of space, which made it necessary for him to attribute to it a quite definite state of motion, which yet did not appear to be fully determined by the phenomena of mechanics. This space was conceived as absolute in another sense also; its inertia-determining effect was conceived as autonomous, i.e., not to be influenced by any physical circumstance whatever; it affected masses but nothing affected it.

And yet in the minds of physicists space remained until the most recent time simply the passive container of all events, playing no part in physical happenings itself. Thought only began to take a new turn with the wave theory of light and the theory of the electro-magnetic field of Faraday and Clerk Maxwell. It became clear that there existed in free space conditions which propagated themselves in waves, as well as localised fields which were able to exert force on electrical masses or magnetic poles brought to the spot. Since it would have seemed utterly absurd to the physicists of the nineteenth century to attribute physical functions or states to space itself, they invented a medium pervading the whole space, on the model of ponderable matter — the ether, which was supposed to act as a vehicle for electro-magnetic phenomena, and hence for those of light also. The states of this medium, imagined as constituting the electro-magnetic fields, were at first thought of mechanically, on the model of the elastic deformations of rigid ~~bodo~~dies. But this

mechanical theory of the ether was never quite successful.
So the idea of a closer explanation of the nature of the electric fields was given up. The ether thus became a kind of matter, whose only function was to act as a substratum for electrical fields which were by their very nature not further analysable. The picture was, then, as follows:— Space is filled by the ether, in which the material corpuscles & atoms of ponderable matter swim; the atomic structure of the latter had been securely established by the turn of the century.

Since the reciprocal action of bodies was supposed to be accomplished through fields, there had also to be a gravitational field in the ether, whose field-law had, however, assumed no clear form at that time. The ether was only accepted as the seat of all operations of force which make themselves effective across space. Since it had been realised that electrical masses in motion produce a magnetic field, whose energy acted as a model for inertia, inertia also appeared as a field-action localised in the ether.

The mechanical properties of the ether were at first a mystery. Then came H. A. Lorentz's great discovery. All the phenomena of electro-magnetism then known could be explained on the basis of two assumptions: that the ether is firmly fixed in space — that is to say, unable to move at all; and that electricity is firmly lodged in the mobile elementary particles. To-day his discovery may be expressed as follows:— Physical space and the ether are only different terms for the same thing; fields are physical conditions of space. For if no particular state of motion can be ascribed to the ether, there does not seem to be any ground for introducing it as an entity of a special sort alongside of

14. But the physicists were still far removed from such a way of thinking; space was still, for them, a rigid, homogeneous something, susceptible of no change or conditions. Only the genius of Riemann, solitary and uncomprehended, had already won its way by the middle of last century to a new conception of space, in which it was deprived of its rigidity and its power to take part in physical events recognised as possible. This intellectual achievement commands our admiration all the more for having preceded Faraday's and Clerk Maxwell's field-theory of electricity. Then came the special theory of relativity with its recognition of the physical equivalence of all inertial systems. The inseparableness of time and space emerged in connection with electrodynamics or the law of the propagation of light. Hitherto it had been silently assumed that the four-dimensional continuum of events could be split up into time and space in an objective manner — i.e., that an absolute significance attached to the "now" in the world of events. With the discovery of the relativity of simultaneity, space and time were merged in a single continuum in the same way as the three dimensions of space had been before. Physical space was thus increased to a four-dimensional space which also included the dimention of time. The four-dimensional space of the special theory of relativity is just as rigid and absolute as Newton's space.

The theory of relativity admirably exemplifies the fundamental character of the modern development of theoretical science. The hypotheses with which it starts become

steadily more abstract and remote from experience. On the other hand it gets nearer to the grand aim of all science, which is to cover the greatest possible number of empirical facts by logical deduction from the smallest possible number of hypotheses or axioms. Meanwhile the train of thought leading from the axioms to the empirical facts or verifiable consequences gets steadily longer and more subtle. The theoretical scientist is compelled in an increasing degree to be guided by purely mathematical, formal considerations in his search for a theory, because the physical experience of the experimentor cannot lift him into the regions of highest abstraction. The predominantly inductive methods appropriate to the youth of science are giving place to tentative deduction. Such a theoretical structure needs to be very thoroughly elaborated before it can lead to conclusions which can be compared with experience. Here, too, the observed fact is undoubtedly the supreme arbiter; but it cannot pronounce sentence untill the wide chasm separating the axioms from their verifiable consequences has been bridged by much intense hard thinking. The theorist has to set about this Herculean task in the clear consciousness that his efforts may only be destined to deal the death-blow to his theory. The theorist who undertakes such a labour should not be carped at as "fanciful"; on the contrary, he should be encouraged to give free reign to his fancy, for there is no other way to the goal. His is no idle day-dreaming, but a search for the logically simplest possibilities and their consequences. This plea was needed in order to make the hearer or reader more ready to follow the ensuing train of ideas with attention; it is the line of thought which has led from the special to the

-ral theory of relativity and thence to its latest offshoot, the unitary field-theory. In this exposition the use of mathematical symbols cannot be avoided.

We start with the special theory of relativity. This theory is still based directly on an empirical law, that of the constant velocity of light. Let P be a point in empty space, P' one separated from it by a length $d\sigma$ and infinitely near it. Let a flash of light be emitted from P at a time t and reach P' at a time $t+dt$. Then,

$$d\sigma^2 = c^2 dt^2$$

If dx_1, dx_2, dx_3 are the orthogonal projections of $d\sigma$, and the imaginary time co-ordinate $\sqrt{-1}\ ct = x_4$ is introduced, then the above mentioned law of the constancy of the propagation of light takes the form

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 = 0\ .$$

Since this formula expresses a real situation, we may attribute a real meaning to the quantity ds, even supposing the neighbouring points of the four-dimensional continuum are selected in such a way that the ds belonging to them does not disappear. This is more or less expressed by saying that the four-dimensional space (with imaginary time-co-ordinate) of the special theory of relativity possesses a Euclidean metric.

The fact that such a metric is called Euclidean is connected with the following. The position of such a metric in a three-dimensional continuum is fully equivalent to the positing of the axioms of Euclidean geometry. The defining equation of the metric is thus nothing but the Pythagorean theorem applied to the differentials of the co-ordinates.

Such alteration of the co-ordinates (by transformation)

is permitted in the special theory of relativity, since in the new co-ordinates too the magnitude ds^2 (fundamental invariant) is expressed in the new differentials of the co-ordinates by the sum of the squares. Such transformations are called Lorentz transformations.

The heuristic method of the special theory of relativity is characterised by the following principle:— Only those equations are admissible as an expression of natural laws which do not change their form when the co-ordinates are changed by means of a Lorentz transformation (co-variance of equations in relation to Lorentz transformations).

This method led to the discovery of the necessary connection between impulse and energy, the strength of an electric and a magnetic field, electrostatic and electro-dynamic forces, inert mass and energy; and the number of independent concepts and fundamental equations was thereby reduced.

This method pointed beyond itself. It is true that the equations which express natural laws are co-variant in relation to Lorentz transformations only and not in relation to other transformations? Well, formulated in that way the equation really means nothing, since every system of equations can be expressed in general co-ordinates. We must ask, Are not the laws of nature so constituted that they receive no real simplification through the choice of any one particular set of co-ordinates?

We will only mention in passing that our empirical principle of the equality of inert and heavy masses prompts us to answer this question in the affirmative. If we elevate the equivalence of all co-ordinate systems for the formulation of natural laws into a principle, we arrive at the general

... of relativity, provided we stick to the law of the constant velocity of light or to the hypothesis of the objective significance of the Euclidean metric at least for infinitely small portions of four-dimensional space.

This means that for finite regions of space the existence (significant for physics) of a general Riemannian metric is presupposed according to the formula

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} \, dx^\mu \, dx^\nu ,$$

whereby the summation is to be extended to all index combinations from 11 to 44.

The structure of such a space differs absolutely radically in one respect from that of a Euclidean space. The coefficients $g_{\mu\nu}$ are for the time being any functions whatever of the co-ordinates x_1 to x_4, and the structure of the space is not really determined untill these functions $g_{\mu\nu}$ are really known. It is only determined more closely by specifying laws which the metrical field of the $g_{\mu\nu}$ satisfies. On physical grounds this gave rise to the conviction that the metrical field was at the same time the gravitational field.

Since the gravitational field is determined by the configuration of masses and changes with it, the geometric structure of this space is also dependent on physical factors. Thus according to this theory space is — exactly as Riemann guessed — no longer absolute; its structure depends on physical influences. Physical geometry is no longer an isolated self-contained science like the geometry of Euclid.

The problem of gravitation was thus reduced to a mathematical problem: it was required to find the simplest fundamental equations which are co-variant ...

relation to any transformation of co-ordinates whatever.

I will not speak here of the way this theory has been confirmed by experience, but explain at once why Theory could not rest permanently satisfied with this success. Gravitation had indeed been traced to the structure of space, but besides the gravitational field there is also the electro-magnetic field. This had, to begin with, to be introduced into the theory as an entity independent of gravitation. Additional terms which took account of the existence of the electro-magnetic field had to be included in the fundamental equations for the field. But the idea that there were two structures of space independent of each other, the metric-gravitational and the electro-magnetic, was intolerable to the theoretical spirit. We are forced to the belief that both sorts of field must correspond to a unified structure of space.

The "unitary field-theory" which represents itself as a mathematically independent extension of the general theory of relativity, attempts to fulfil this last postulate of the field theory. The formal problem should be put as follows:— Is there a theory of the continuum in which a new structural element appears side by side with the metric such that it forms a single whole together with the metric? If so, what are the simplest field-laws to which such a continuum can be made subject? And finally, are these field-laws well fitted to represent the properties of the gravitational field and the electro-magnetic field? Then there is the further question whether the corpuscles (electrons and protons) can be regarded as locations of particularly dense fields, whose movements are determined by the field equations. At present there only one way

in answering the first three questions. The space structure on which it is based may be described as follows, and the description applies equally to a space of any number of dimensions.

Space has a Riemannian metric. This means that the Euclidean geometry holds good in the infinitesimal neighbourhood of every point P. Thus for the neighbourhood of every point P there is a local Cartesian system of co-ordinates, in reference to which the metric is calculated according to the Pythagorean theorem. If we now imagine the length l cut off from the positive axes of these local systems, we get the orthogonal local unit vector. Such a local unit vector is to be found in every other point P' of space also. Thus, if a linear element (PG or P'G') starting from the points P or P', is given, then the magnitude of this linear element can be calculated by the aid of the relevant local ~~vector~~ unit vector, from its local co-ordinates by means of Pythagoras's theorem. There is therefore a definite meaning in speaking of the numerical equality of the linear elements PG and P'G'.

It is essential to observe now that the local orthogonal unit vectors are not completely determined by the metric. For we can still select the orientation of the unit vectors perfectly freely without causing any alteration in the result of calculating the size of the linear element according to Pythagoras's theorem. A corollary of this is that in a space whose structure consists exclusively of a Riemannian metric, two linear elements PG and P'G' can be compared with regard to their magnitude but not their direction; in particular, there is no sort of point in saying that the two linear elements are parallel to one another. In this respect, therefore, the purely metrical (Riemannian) space is

less rich in structure than the Euclidean.

Since we are looking for a space which exceeds Riemannian space in wealth of structure, the obvious thing is to enrich Riemannian space by adding the relation of direction or parallelism. Therefore for every direction through P let there be a definite direction through P', and let this mutual relation be a determinate one. We call the directions thus related to each other "parallel". Let this parallel relation further fulfil the condition of angular uniformity: if PG and PK are two directions in P, P'G' and P'K' the corresponding parallel directions through P', then the angles KPG and K'P'G' (measurable on Euclidean lines in the local system) should be equal.

The basic space-structure is thereby completely defined. It is most easily described mathematically as follows:— In the definite point P we suppose an orthogonal unit vector with definite, freely chosen orientation. In every other point P' of space we so orient its local unit vector that its axes are parallel to the corresponding axes at the point P. Given the above structure of space and free choice in the orientation of the unit vector at one point P, all unit vectors are thereby completely defined. In the space P let us now imagine any Gaussian system of co-ordinates and that in every point the axes of the unit vector there are projected on to it. This system of n^2 components completely describes the structure of space.

This spatial structure stands, in a sense, midway between the Riemannian and the Euclidian. In contrast to the former, it has room for the straight line, that is to say a line all of whose elements are parallel to each other in pairs. The geometry here described differs from the Euclidian in the non-existence of the parallelogram.

If at the ends P and G of a length PG two equal and parallel lengths PP' and GG' are marked off, P'G' is in general neither equal nor parallel to PG.

The mathematical problem now solved so far is this:— What are the simplest conditions to which a space-structure of the kind described can be subjected? The chief question which still remains to be investigated is this:— To what extent can physical fields and primary entities be represented by solutions free from singularities, of the equations which answer the former question

Notes on the origin of the general theory of Relativity.

I gladly accede to the request that I should say something about the history of my own scientific work. Not that I have an exaggerated notion of the importance of my own efforts, but to write the history of other men's work demands a degree of absorption in other people's ideas which is much more in the line of the trained historian; to throw light on one's own earlier thinking appears incomparably easier. Here one has an immense pull over everybody else, and one ought not to leave the opportunity unused out of modesty.

When, by the special theory of relativity, I had arrived at the equivalence of all so-called inertial systems for the formulation of natural laws (1905), the question whether there was not a further equivalence of co-ordinate systems followed naturally, to say the least of it. To put it in another way, if only a relative meaning can be attached to the concept of velocity, ought we nevertheless to persevere in treating acceleration as an absolute concept?

From the purely kinematic point of view there was

no doubt about the relativity of all motion whatever; but physically speaking, the inertial system seemed to occupy a privileged position which made the use of co-ordinate systems moving in other ways appear artificial.

I was, of course, acquainted with Mach's view, according to which it appeared conceivable that what inertial resistance counteracts is not acceleration as such but acceleration with respect to the masses of the other bodies existing in the world. There was something fascinating about this idea to me, but it provided no workable basis for a new theory.

I first came a step nearer to the solution of the problem when I attempted to deal with the law of gravity within the framework of the special theory of relativity. Like most writers at the time, I tried to frame a field-law for gravitation, since it was no longer possible, at least in any natural way, to introduce direct action at a distance, owing to the abolition of the notion of absolute simultaneity.

The simplest thing was, of course, to retain the Laplacian scalar potential of gravity, and to complete the equation of Poisson in an obvious manner by a term differentiated as to time in such a way that the special theory of relativity was satisfied. The law of motion of the mass point in a gravitational field had also to be adapted to the special theory of relativity. The path was not so unmistakably ~~pointed~~ marked out here, since the inert mass of a body might depend on the gravitational potential. In fact this was to be expected on account of the principle of the inertia of energy.

These investigations, however, led to a result which raised my strong suspicions. According to classical mechanics the vertical acceleration of a body in the vertical gravitational field is independent of the horizontal component of

velocity. Hence in such a gravitational field the vertical accelera-
tion of a mechanical system or of its centre of gravity w[…]
out independently of its internal kinetic energy. But in [my]
theory I advanced the acceleration of a falling body was not
independent of the horizontal velocity or the internal energy of a system.

This did not fit in with the old experimental fact that all bodies have the same acceleration in a gravitational field. This law, which may also be formulated as the law of the equality of inertial and gravitational mass, was now brought home to me in all its significance. I was in the highest degree amazed at its persistence and guessed that in it must lie the key to a deeper understanding of inertia and gravitation. I had no serious doubts about its strict validity, even without knowing the results of the admirable experiments of Eötvös, which — if my memory is right — I only came to know later.

I now abandoned as inadequate the attempt to treat the problem of gravitation, in the manner outlined above, within the framework of the special theory of relativity. It clearly failed to do justice to the most fundamental property of gravitation. The principle of the equality of inertial and gravitational mass could now be formulated quite clearly as follows :— In a homogeneous gravitational field all motions take place in the same way as in the absence of a gravitational field in relation to a uniformly accelerated co-ordinate system. If this principle held good for any events whatever (the "principle of equivalence"), this was an indication that the principle of relativity needed to be extended to co-ordinate systems in non-uniform motion with respect to each other, if we were to reach an easy and natural theory of the gravitational field. Such reflections kept me busy from 1908 to 1911, and I attempted to draw special conclusions from them, of which I do not propose to speak here. For the moment the one important thing was the discovery that a reasonable theory of gravitation could

only be hoped-for from an extension of the principle of relativity.

What was needed, therefore, was to frame a theory whose equations kept their form in the case of non-linear transformations of the co-ordinates. Whether this was to apply to absolutely any (constant) transformations of co-ordinates or only to certain ones, I could not for the moment say.

I soon saw that bringing in non-linear transformations as the principle of equivalence demanded, was inevitably fatal to the simple physical interpretation of the co-ordinates — i.e., that it could no longer be required that differentials of co-ordinates should signify direct results of measurement with ideal scales or clocks. I was much bothered by this piece of knowledge, for it took me a long time to see what co-ordinates in general really meant in physics. I did not find the way out of this dilemma till 1912, and then it came to me as a result of the following consideration: —

A new formulation of the law of inertia had to be found which in case of the absence of a real "gravitational field with application of an inertial system" as a co-ordinate system passed over into Galileo's formula for the principle of inertia. The latter amounts to this: — A material point, which is acted on by no force, will be represented in four-dimensional space by a straight line that is to say, by a line that is as short as possible or, more correctly, an extreme line. This concept presupposes that of the length of a linear element, that is to say, a metric. In the special theory of relativity, as Minkowski had shown, this metric was a quasi-Euclidean one, i.e., the square of the "length" ds of the linear

element was a definite quadratic function of the differentials of the co-ordinates.

If other co-ordinates are introduced by means of a non-linear transformation, ds^2 remains a homogeneous function of the differentials of the co-ordinates, but the coefficients of this function ($g_{\mu\nu}$) cease to be constant and become certain functions of the co-ordinates. In mathematical terms this means that physical (four-dimensional) space has a Riemannian metric. The time-like extremal lines of this metric furnish the law of motion of a material point which is acted on by no force apart from the forces of gravity. The coefficients ($g_{\mu\nu}$) of this metric at the same time describe the gravitational field with reference to the co-ordinate system selected. A natural formulation of the principle of equivalence had thus been found, the extension of which to any gravitational field whatever formed a perfectly natural hypothesis.

The solution of the above-mentioned dilemma was therefore as follows:— A physical significance attaches not to the differentials of the co-ordinates but only to the Riemannian metric co-ordinated with them. A workable basis had now been found for the general theory of relativity. Two further problems remained to be solved, however.

(1) If a field-law is given in terminology of the special theory of relativity, how can it be transferred to the case of a Riemannian metric?

(2). What are the differential laws which determine the Riemannian metric (i.e., $g_{\mu\nu}$) itself?

I worked out on these problems from 1912 to 1914 together with my friend Grossmann. We found that the mathematical methods for solving problem (1) lay ready to our hands in the infinitesimal differential calculus of Ricci and Levi-Civita.

As for problem (2), its solution obviously needed invariant differential systems of the second order taken from $g_{\mu\nu}$. We soon saw that these had already been established by Riemann (the tensor of curvature). We had already considered the right field equations for gravitation two years before the publication of the general theory of relativity, but we were unable to see how they could be used in physics. On the contrary, I felt sure that they could not do justice to experience. Moreover I believed that I could show on general considerations that a law of gravitation invariant in relation to any transformation of co-ordinates whatever was inconsistent with the principle of causation. There were errors of thought which cost me two years of excessively hard work, untill I finally recognised them as such at the end of 1915 and succeeded in linking up with the facts of astronomical experience, after having ruefully returned to the Riemannian curvature.

In the light of knowledge attained, the happy achievement seems almost a matter of course, and any intelligent student can grasp it without too much trouble. But the years of anxious searching in the dark, with their intense longing, their alternations of confidence and exhaustion, and the final emergence into the light, — only those who have experienced it can understand that.

Relativity and the Ether.

Why is it that alongside of the notion, derived by abstraction from everyday life, of ponderable matter physicists have set the notion of the existence of another sort of matter, the ether? The reason lies no doubt in those phenomena which gave rise to the theory of forces acting at a distance, and in those properties of light which led to the wave-theory. Let us shortly consider these two things.

Non-physical thought knows nothing of forces acting at a distance. When we try to explain our experiences of bodies by a complete causal scheme, there seems at first sight to be no reciprocal interaction except what is produced by means of immediate contact, eg., the transmission of motion by impact, pressure or pull, heating or inducing combustion by means of a flame, etc. To be sure, gravity, that is to say, a force acting at a distance, does play an important part in everyday experience. But since the gravity of bodies presents itself to us in common life as something constant, dependent on no variable temporal or spatial cause, we do not ordinarily think of any cause in connection with it and thus are not conscious of its character as a force acting at a distance. It was not till Newton's theory of gravitation that a cause was assigned to it; it was then explained as a force acting at a distance, due to mass. Newton's theory certainly marks the greatest step ever taken in linking up natural phenomena causally. And yet his contemporaries were by no means satisfied with it, because it seemed to contradict the principle derived from the rest of experience, that reciprocal action only takes place by means of direct contact, not by direct action at a distance,

without any means of transmission.

Man's thirst for knowledge only acquiesces in such a dualism reluctantly. How could unity in our conception of natural forces be saved? People could either attempt to treat the forces which appear to us to act by contact as acting at a distance, though only making themselves felt at very small distances; this was the way generally chosen by Newton's successors, who were completely under the spell of his teaching. Or they could take the line that Newton's forces acting at a distance only appeared to act thus directly; that they were really transmitted by a medium which permeated space, either by motions or by an elastic deformation of this medium. Thus the desire for unity in our view of the nature of these forces led the hypothesis of the ether. It certainly led to no advance in the theory of gravitation or in physics generally to begin with, so that people got into the habit of treating Newton's law of force as an irreducible axiom. But the ether hypothesis was bound always to play a part, even if it was mostly a latent one at first, in the thinking of physicists.

When the extensive similarity which exists between the properties of light and those of the elastic waves in ponderable bodies was revealed in the first half of the nineteenth century, the ether hypothesis acquired a new support. It seemed beyond a doubt that light was to be explained as the vibration of an elastic, inert medium filling the whole space. It also seemed to follow necessarily from the polarisability of light that this medium, the ether, must be of the nature of a solid body, because transverse waves are only possible in such a body and not in a fluid. This inevitably led to the theory of the "quasi-rigid" luminiferous ether, whose parts are incapable of any motion

with respect to each other beyond the small deformations which correspond to the waves of light.

This theory, also called the theory of the stationary luminiferous ether, derived strong support from the experiments, of fundamental importance for the special theory of relativity too, of Fizeau, which proved conclusively that the luminiferous ether does not participate in the motions of bodies. The phenomena of aberration also lent support to the theory of the quasi-rigid ether.

The evolution of electrical theory along the lines laid down by Clerk Maxwell and Lorentz gave a most peculiar and unexpected turn to the development of our ideas about the ether. For Clerk Maxwell himself the ether was still an entity with purely mechanical properties, though of a far more complicated kind than those of tangible solid bodies. But neither ~~m~~ Maxwell nor his successors succeeded in thinking out a mechanical model for the ether capable of providing a satisfactory mechanical interpretation of Maxwell's laws of the electro-dynamic field. The laws were clear and simple, the mechanical interpretations clumsy and contradictory. Almost imperceptibly theoretical physicists adapted themselves to this state of affairs (which was a most depressing one from the point of view of their mechanistic programme), especially under the influence of the electro-dynamic researches of Heinrich Hertz. Whereas they had formerly demanded of an ultimate theory that it should be based upon fundamental concepts of a purely mechanical kind (e.g., mass-densities, velocities, deformations, forces of gravitation), they gradually became accustomed to admitting electric and magnetic field-strengths as fundamental concepts alongside of the

mechanical ones, without insisting upon a mechanical interpretation of them. The purely mechanistic view of nature was thus abandoned. This change led to a dualism in the sphere of fundamental concepts which was in the long run intolerable. To escape from it, the converse attempt was made to reduce mechanical concepts to electrical ones. The experiments with β-rays and high velocity cathode rays did much to shake confidence in the strict validity of Newton's mechanical equations.

Heinrich Hertz took no steps towards mitigating this dualism. Matter appears in his work as the substratum not only of velocities, kinetic energy, and mechanical forces of gravity, but also of electro-magnetic fields. Since such fields are also found in a vaccum — i.e., in unoccupied ether — the ether also appears as the substratum of electro-magnetic fields, entirely similar in nature to ponderable matter and ranking alongside it. In the presence of matter it shares in the motions of the latter and has a velocity everywhere in empty space; the ether velocity nowhere changes discontinuously. There is no fundamental distinction between the Hertzian ether and ponderable matter, which partly consists of ether.

Hertz's theory not only suffered from the defect that it attributed to matter and the ether both mechanical and electrical properties, with no rational connection between them; it was also inconsistent with the result of Fizeau's famous experiment on the velocity of the propagation of light in a liquid in motion and other well-authenticated empirical facts.

Such was the position when H.A. Lorentz entered the field.

Lorentz brought theory into harmony with experiment, a[nd] did it by a marvellous simplification of basic concepts. [He] achieved this advance in the science of electricity, the most important since Clerk Maxwell, by divesting the ether of its mechanical, matter of its electro-magnetic properties. Inside material bodies no less than in empty space the ether alone, no[t] atomically conceived matter, is the seat of electro-magnetic fields. According to Lorentz the elementary particles of matter are only capable of executing movements; their electro-magne[tic] activity is entirely due to the fact that they carry electric char[ges]. Lorentz thus succeeded in reducing all electro-magnetic phenomena to Maxwell's equations for a field in vacuo.

As regards the mechanical nature of Lorentz's ether, one might say of it, with a touch of humour, that immobility was the only mechanical property which Lorentz ~~so~~ left it. It may be added that the whole difference which the special theory of relativity made in our conception of the ether lay in this, that it divested the ether of its last mechanical quality, namely immobility. How this is to be understood I will explain immediately.

The Maxwell-Lorentz theory of the electro-magnetic field served as the model for the space-time theory and the kinematics of the special theory of relativity. Hence it satisfie[s] the conditions of the special theory of relativity; but looked-at from the standpoint of the latter, it takes on a new aspect. If C is a co-ordinate system in respect to which the Lorentzian ether is at rest the Maxwell-Lorentz equations hold good first of all in regard to C. According to the special theory of relativity these same

equations hold good in exactly the same sense in regard to any new co-ordinate system C', which is in uniform translatory motion with respect to C. We are now faced with awkward question why the system C, which is physically perfectly equivalent to system C', should be distinguished from the latter by assuming that the ether is ~~in respect~~ at rest in respect to it. Such an asymmetry of the theoretical structure, to which there is no corresponding asymmetry in the system of empirical facts, is intolerable to the theorist. In my view the physical equivalence of C and C' with the assumption that the ether is at rest in respect to C but in motion with respect to C', though not absolutely wrong from a logical point of view, is nevertheless unsatisfactory.

The most obvious line to adopt in the face of this situation seemed to be the following:— There is no such thing as the ether. The electro-magnetic fields are not states of a medium but independent realities, which cannot be reduced to terms of anything else and are bound to no substratum, any more than are the atoms of ponderable matter. This view is rendered the more natural by the fact that, according to Lorentz's theory, electro-magnetic radiation carries impulse and energy like ponderable matter, and that matter and radiation, according to the special theory of relativity, are both of them only particular forms of distributed energy, inasmuch as ponderable mass loses its exceptional position and merely appears as a particular form of energy.

In the meantime more exact Reflection shows that this denial of the existence of the ether is not demanded by the restrictio

principle of relativity. We can assume the existence of an ether; but we must abstain from ascribing a definite state of motion to it, i.e. we must divest it by abstraction of the last mechanical characteristic which Lorentz left to it. We shall see later on that this way of looking at it, the intellectual possibility of which I shall try to make clearer by a comparison that does not quite go on all fours, is justified by the results of the general theory of relativity.

Consider waves on the surface of water. There are two quite different things about this phenomenon which may be described. One can trace the successive changes which take place in the undulating surface where the water and the air meet. One can also — with the aid of small floating bodies, say — trace the successive positions of the individual particles. If there were in the nature of the case no such floating bodies to aid us in tracing the movement of the particles of the liquid, if nothing at all could be observed in the whole procedure except the fleeting changes in the position of the space occupied by the water, we should have no ground for supposing that the water consists of particles. But we could none the less call it a medium.

Something of the same sort confronts us in the electro-magnetic field. We may conceive the field as consisting of lines of force. If we try to think of these lines of force as some-thing material in the ordinary sense of the word, there is a temp-tation to ascribe the dynamic phenomena involved in their motion, each single line being followed out through time. It is, however, well known that this way of looking at the matter leads to contradictions.

generalising, we must say that we can conceive of extended physical objects to which the concept of motion cannot be applied. They must not be thought of as consisting of particles, whose course can be followed out separately through time. In the language of Minkowski this is expressed as follows:– Not every extended entity in the four-dimensional world can be regarded as composed of world-lines. The special principle of relativity forbids us to regard the ether as composed of particles, the movements of which can be followed out through time, but the theory is not incompatible with ether hypothesis as such. Only we must take care not to ascribe a state of motion to the ether.

From the point of view of the special theory of relativity the ether hypothesis does certainly seem an empty one at first sight. In the equations of an electro-magnetic field, apart from the density of the electrical charge, nothing appears except the strength of the field. The course of electro-magnetic events in a vacuum seems to be completely determined by that inner law, and independent of other physical quantities. The electro-magnetic field seems to be the final irreducible reality, and it seems superfluous at first sight to postulate a homogeneous, isotropic etheric medium, of which these fields are to be considered as states.

On the other hand, there is an important argument in favour of the ether hypothesis. To deny the existence of the ether is, in the last analysis, to deny all physical properties to empty space. But a view is inconsistent with the fundamental facts of mechanics. The mechanical behaviour of a corporeal system floating freely in empty space depends not only on the relative positions (intervals) and velocities of its masses, but also on its state of rotation, which cannot be regarded, physically as a property belonging to the system as such. In order

be able to regard the rotation of a system at least formally as something real, Newton regarded space as objective. Since he treats his absolute space as a real thing, rotation with respect to absolute space is also something real to him. Newton could equally well have called his absolute space "the ether"; all that matters is that another and imperceptible entity, in addition to observable objects, has to be regarded as real, in order that acceleration, or rotation, may be regarded as real.

Mach did indeed try to avoid the necessity for postulating an imperceptible real entity, by substituting in mechanics a mean velocity with respect to the totality of masses in the world for acceleration with respect to absolute space. But inertial resistance with respect to the relative acceleration of distant masses presupposes direct action at a distance. Since the modern physicist does not consider himself entitled to assume that, this view brings him back to the ether, which has to act as the medium of inertial action. This conception of the ether, to which Mach's approach leads, differs in important respects from that of Newton, Fresnel and Lorentz. Mach's ether not only conditions the behaviour of inert masses but is also conditioned, as regards its state, by them.

Mach's notion finds its full development in the ether of the general theory of relativity. According to this theory the metrical properties of the space-time continuum in the neighbourhood of separate space-time points are different and conjointly conditioned by matter existing outside the region in question. This spatio-temporal variability of the relations of scales and clocks to each other, or the knowledge that "empty space" is, physically speaking

neither homogeneous nor isotropic", which [illegible] describe its state by means of ten functions, the gravitational potentials $g_{\mu\nu}$, has no doubt finally disposed of the notion that space is physically empty. But this has also once more given the ether notion a definite content — though one very different from that of the ether of the mechanical wave-theory of light. The ether of the general theory of relativity is a medium which is itself free of all mechanical and kinematic properties, but helps to determine mechanical (and electro-magnetic) events.

The radical novelty in the ether of the general theory of relativity as against the ether of Lorentz lies in this, that the state of the former at every point is determined by the laws of its relationship with matter and with the state of the ether at neighbouring points, expressed in the form of differential equations, whereas the state of Lorentz's ether, in the absence of electro-magnetic fields, is determined by nothing outside it and is the same everywhere. The ether as conceived by the general theory of relativity passes over into Lorentz's if constants are substituted for the spatial functions describing its state, thus neglecting the causes conditioning the latter. One may therefore say that the ether of the general theory of relativity is derived by relativisation from the ether of Lorentz.

The part which the new ether is destined to play in the physical scheme of the future is still a matter of uncertainty. We know that it determines both material relations in the space-time continuum, e.g., the possible configurations of solid bodies, as gravitational fields; but we do not know whether it plays

part in the structure of the electric particles which constitute matter. Nor do we know whether its structure only differs materially from that of Lorentz's in the proximity of ponderable masses, whether, in fact, the geometry of spaces of cosmic extent is, taken as a whole, almost Euclidean. We can, however, maintain on the strength of the relativistic equations of gravitation that spaces of cosmic proportions must depart from Euclidean behaviour if there is a positive mean density of matter, however small, in the universe. In this case the Universe must necessarily form a closed space of finite size, this size being determined by the value of the mean density of matter.

If we consider the gravitational field and the electro-magnetic field from the standpoint of the ether hypothesis, we find a notable fundamental difference between the two. No space and no portion of space is without gravitational potential; for this gives it its metrical properties without which it is not thinkable at all. The existence of the gravitational field is directly bound up with the existence of space. On the other hand, a portion of space without an electro-magnetic field is perfectly conceivable; hence the electro-magnetic field, in contrast to the gravitational field, seems in a sense to be connected with the ether only in a secondary way, since its formal nature is by no means determined by the gravitational ether. In the present state of theory it looks as if the electro-magnetic field, as compared with the gravitational field, were based on a completely new formal motive; as if nature, instead of endowing the gravitational ether with fields of the electro-magnetic type, might equally well have endowed it with fields of a quite different type, for example fields with a scalar potential

It would, of course, be a great step forward if we succeeded in
combining the gravitational field and the electro-magnetic field into
a single structure. Only so could the era in theoretical physics ina
gurated by Faraday and Clerk Maxwell be brought to a satisfactory cl

The antithesis of ether and matter would then fade away
and the whole of physics would become a completely enclosed inte
llectual system, like geometry, kinematics and the theory of gravi
tation, through the general theory of relativity. An exceedingly brilli
ant attempt in this direction has been made by the mathematician
H. Weyl; but I dont think that it will stand the test of reality. More
in thinking about the immediate future of theoretical physics we
cannot unconditionally dismiss the possibility that the facts summa
rized in the quantum theory may set impassable limits to the field-theo

We may sum up as follows:- According to the general
theory of relativity space is endowed with physical qualities;
in this sense, therefore, an ether exists. Spa c without an ether
inconceivable. For in such a space there would not only
propagation of light, but no possibility of the existence of
and clocks, and therefore no spatio-temporal distances
the physical sense. But this ether must not be thought
endowed with the properties characteristic of pondera
as composed of particles the motion of which can be follo
nor may the concept of motion be applied to it.